Equations of Motion
Equations of Motion

- Vectors and scalars
- Relative velocity
- Calculating velocity
- Calculating displacement
- Summary activities
**What is a scalar?**

**Scalar** quantities are measured with numbers and units.

- **length** (e.g. 16 cm)
- **temperature** (e.g. 102°C)
- **time** (e.g. 7 s)
**What is a vector?**

Vector quantities are measured with numbers and units, but also have a specific **direction**.

- **acceleration** (e.g. 30 m/s² upwards)
- **displacement** (e.g. 200 miles north-west)
- **force** (e.g. 2 N downwards)
**Distance** is a measure of the total ground covered between start and end points, and depends on the route taken.

In a car, distance is measured by a millimeter.
Speed or velocity?

Distance is a scalar and displacement is a vector. Similarly, **speed is a scalar** and **velocity is a vector**.

**Speed** is the rate of change of **distance** in the direction of travel. Speedometers in cars measure speed. Direction does not matter.

**Velocity** is a rate of change of **displacement** and has both **magnitude** and **direction**.

Averages of both can be useful:

\[
\text{average speed} = \frac{\text{distance}}{\text{time}}
\]

\[
\text{average velocity} = \frac{\text{displacement}}{\text{time}}
\]
Vector or scalar?

Are these quantities scalars or vectors?

- Scalar
- Vector

friction

solve
Adding perpendicular vectors

Displacement is a quantity that is independent of the route taken between start and end points.

If a car moves from A to C, first by travelling north to B and then east to C, its total displacement will be the same as if it had just moved north-east in a straight line from A to C.

Two or more vectors can be added ‘nose to tail’ to calculate a resultant vector.

Any two vectors of the same type can be added in this way to find a resultant.
Understanding vector calculations

What is missing from each vector equation?

Equation 2 of 3

6, east + 6, north + 6, west = A

6, south
6, north
6, north-west
6, north-east
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Relative velocities

If two trains are travelling at 25 m/s on parallel tracks, but in opposite directions, do they have the same velocity?

No – velocity is a measure of direction as well as speed.

How are these different velocities written?

Train A has a velocity of \(25\text{ m/s}\).

Train B has a velocity of \(-25\text{ m/s}\).
Relative velocities

If two trains are travelling at 25 m/s on parallel tracks, what is their relative velocity?

Train A has a velocity of 25 m/s.
Train B has a velocity of –25 m/s.

The velocity of Train A relative to Train B is:

\[
25 \text{ m/s} - (-25 \text{ m/s}) = 50 \text{ m/s}
\]
See if you can determine the relative velocities of two trains.

Press **start** to begin.
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Motion under constant acceleration

Calculating vector quantities such as velocity or displacement can be complicated, but when acceleration is constant, four equations always apply.

\[ v = u + at \]
\[ s = \frac{1}{2}(u + v)t \]
\[ v^2 = u^2 + 2as \]
\[ s = ut + \frac{1}{2}at^2 \]

These are sometimes known as the **constant acceleration equations**, or the ‘uvast’ or ‘suvat’ equations.

What do the symbols \( u, v, a, s \) and \( t \) represent?
Representing motion

The symbol \( a \) represents acceleration.
The symbol \( t \) represents time.

Displacement, \( s \) is always measured relative to a starting position, so it is always true that when \( t = 0 \), \( s = 0 \).

Velocity at time \( t \) is represented by \( v \), and \( u \) represents the value of \( v \) when \( t = 0 \). This is the initial velocity.

- \( a \) = acceleration
- \( t \) = time
- \( s \) = displacement
- \( v \) = velocity
- \( u \) = initial velocity
Two velocity equations

Two of the constant acceleration equations are velocity equations.

\[ v = u + at \quad \text{and} \quad v^2 = u^2 + 2as \]

The first of these can be used to find the velocity at a particular time \( t \). The second can be used to find the velocity at a particular displacement \( s \).

When deciding which equation to use, it is good practice to write down what you know about the values of \( u \), \( v \), \( a \), \( s \) and \( t \) before you start any calculations.
Calculating final velocity

A cyclist accelerates towards the end of the race in order to win. If he is moving at 6 m/s then accelerates by 1.5 m/s$^2$ for the final five seconds of the race, calculate his speed as he crosses the line.

First write down what you know about $u$, $v$, $a$, $s$ and $t$:

- $u = 6 \text{ m/s}$
- $v = ?$
- $a = 1.5 \text{ m/s}^2$
- $s = $
- $t = 5 \text{ s}$

The question gives a value for $t$. What is the relevant equation?

$$v = u + at$$

$$v = 6 + (1.5 \times 5)$$

$$v = 13.5 \text{ m/s}$$
Calculating initial velocity

A mushroom ejects a spore directly downwards. The spore accelerates at 10 m/s$^2$ and hits the ground 0.1 s later with a final speed of 2 m/s. What is the initial speed of the spore?

First write down what you know about $u$, $v$, $a$, $s$ and $t$:

The question gives a value for $t$. What is the relevant equation?

\[
\begin{align*}
u &= ? \\
v &= 2 \text{ m/s} \\
a &= 10 \text{ m/s}^2 \\
s &= \\
t &= 0.1 \text{ s}
\end{align*}
\]

\[
\begin{align*}
v &= u + at \\
u &= v - at \\
u &= 2 - 10 \times 0.1 \\
v &= 1 \text{ m/s}
\end{align*}
\]
Rearranging a velocity equation

Rearranging equations of motion

Use the equation to find a value for \( v \):

Take the square root of both sides of the equation to find an expression for \( v \):

\[
v = \sqrt{u^2 + 2as}
\]

\[
v^2 = u^2 + 2as
\]
A coin is dropped from a window. If it hits the ground at 10 m/s, work out the height of the window.

First write down what you know about u, v, a, s and t:

\[ u = 0 \text{ m/s} \]
\[ v = 10 \text{ m/s} \]
\[ a = 10 \text{ m/s}^2 \]
\[ s = ? \]
\[ t = \]

The question involves the variables u, v, a and s, so the relevant equation is \( v^2 = u^2 + 2as \).

Start by rearranging the equation to find a formula for s:

\[ s = \frac{v^2 - u^2}{2a} \]
\[ s = \frac{10^2 - 0^2}{2 \times 10} \]
\[ s = \frac{100}{20} \]
\[ s = 5 \text{ m} \]
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Displacement from average speed

One of the constant acceleration equations uses average speed and time to calculate the displacement.

For a initial velocity \( u \) and final velocity \( v \), the average velocity is \( \frac{1}{2}(u + v) \). The displacement is given by multiplying average speed by time:

\[
s = \frac{1}{2}(u + v)t
\]

As with the velocity equations, it is good practice to write down what you know about the values of \( u \), \( v \), \( a \), \( s \) and \( t \) before you attempt any calculations.
A car travelling at 20 m/s takes five seconds to stop. What is the stopping distance of the car?

First write down what you know about u, v, a, s and t:

- \( u = 20 \text{ m/s} \)
- \( v = 0 \text{ m/s} \)
- \( a = \) (unknown)
- \( s = ? \)
- \( t = 5 \text{ s} \)

The question gives u, v and t and asks for a value for s, so the relevant equation is \( s = \frac{1}{2}(u + v)t \):

\[
\begin{align*}
   s &= \frac{1}{2} \times (20 + 0) \times 5 \\
   &= \frac{1}{2} \times 20 \times 5 \\
   &= 50 \text{ m}
\end{align*}
\]
Calculating time to stop

A cat jumps out into the road 34 metres in front of a car. The car is travelling at 30 m/s and begins breaking as soon as it sees the cat, stopping just in time. How long does it take for the car to stop?

First write down what you know about \( u, v, a, s \) and \( t \):

The question gives \( u, v \) and \( t \) and asks for a value for \( s \), so the relevant equation is

\[
s = \frac{1}{2}(u + v)t.
\]

However, \( t \) should be the subject, so it must be rearranged:

\[
t = 2s ÷ (u + v)
\]

\[
t = 2 \times 34 ÷ (30 + 0)
\]

\[
t = 68 ÷ 30
\]

\[
t = 2.27 \text{ s}
\]
Another displacement equation

Displacement can also be found from a known acceleration:

\[ s = ut + \frac{1}{2}at^2 \]

This equation gives displacement as a function of initial velocity, acceleration and time.

As with the other equation, it is good practice to write down what you know about the values of \( u \), \( v \), \( a \), \( s \) and \( t \) before you attempt any calculations.
Rearranging equations of motion

Rearrange the equation to find a formula for $u$:

1. Subtract $\frac{1}{2}at^2$ from both sides.

$$s - \frac{1}{2}at^2 = ut$$

2. Divide both sides by $t$.

$$u = \frac{s - \frac{1}{2}at^2}{t}$$

$$s = ut + \frac{1}{2}at^2$$
Rearranging an equation – calculations

Calculate the acceleration of an ambulance if it starts at rest and takes six seconds to travel 50 m.

First write down what you know about u, v, a, s and t:

| u = 0 m/s | v = |
| a = ? | s = 50 m |
| t = 6 s |

The question involves the variables u, a, s and t, so the relevant equation is \( s = ut + \frac{1}{2}at^2 \).

Start by rearranging the equation to find a formula for \( a \):

\[
a = \frac{2(s - ut)}{t^2} \quad a = \frac{100}{36} \]

\[
a = \frac{2(50 - (0 \times 6))}{6^2} \quad a = 2.8 \text{ m/s}^2
\]
Using the equations of motion

Use your knowledge of motion to answer the following questions.

Press \textbf{"start"} to begin.
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Acceleration – The rate of change of an object's velocity.
Multiple-choice quiz

Will you excel-erate on this Equations of Motion quiz?

Press "start" to begin.

start